Review of *Kronecker Products and Matrix Calculus with Applications*, by Alexander Graham*

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This is a small and specialized book whose purpose is to develop the Kronecker matrix product and the matrix calculus. It assumes a basic knowledge of matrix theory and the multivariable calculus. After a preliminary chapter to introduce notation, the basic properties of the Kronecker product (and Kronecker sum) are derived. These include the product rule and the determination of the eigenvalues of the Kronecker product of an $m \times m$ matrix A and an $n \times n$ matrix B in terms of those of A and B. The proof given for the latter is incomplete, for it is assumed (without saying so) that A and B have m and n linearly independent eigenvectors, respectively. A matrix X when read column by column can be regarded as a vector, $\operatorname{vec} X$. A derivation is given of the permutation matrix, expressed in terms of Kronecker products, which transforms $\operatorname{vec} X$ into $\operatorname{vec} X^t$. Another chapter is concerned with the application of the Kronecker product to the solution of certain matrix equations.

There then follows three chapters on matrix differentiation. The first two are concerned with the differentiation of matrices with respect to a variable (done entrywise) and the derivative of a scalar valued function with respect to a matrix (entrywise again!). Thus, for instance, evaluated are the derivative of the determinant of a matrix Y with respect a matrix X (if X is Y, the result is a matrix of cofactors), the derivative of the nth power of Y with respect to an element of Y, and the derivative of the trace of $Y = X^tAX$ with respect to X (the result is $AX + A^tX$). The main virtue of the introduction of these notions seems to be a compactness of notation rather than a conception of new ideas. The third chapter of this sequence generalizes the preceding two in that the derivative of a matrix Y with respect to a matrix X is defined. The result is a matrix whose entries are the derivatives of each entry of Y with respect to each entry of X. Product rules and chain rules are derived, and some applications are given. For instance, the derivative of X^{-1} with respect to X is obtained.

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The final chapter is concerned with matrix calculus approaches to a least squares problem, the maximum likelihood estimate of the multivariate normal, and the evaluation of Jacobians.

The reviewer found this to be a curious, yet interesting book. Its price ($39.95), however, seems little justified.

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